POLAR MANHATTAN DISPLACEMENT: MEASURING TONAL DISTANCES BETWEEN CHORDS BASED ON INTERVALLIC CONTENT

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Abstract

It would be highly advantageous if an algorithm were able to autonomously describe chords, scales, etc. in a consistent and musically informative way. In this paper, we revisit tonal interval vectors (TIVs), which reveal certain insights as to the interval and tonal nature of pitch class sets. We then describe the qualities and criteria required to comprehensively and consistently measure displacements between TIVs. Next, we present the Polar Manhattan Displacement (PMD), a compound magnitude and phase measure for describing the displacements between pitch class sets in a tonally informed manner. We end by providing examples of how PMD can be used in automated harmonic sequence analysis over a complex chord vocabulary.

Polar Manhattan Displacement (PMD)

PMD is a way of mathematically describing the differences between the tonal properties of groups of notes. Although PMD can be applied to any pitch class set, for the sake of clarity, this presentation focuses specifically on chords.

Chords can be represented as tonal interval vectors (TIVs). A TIV is created by taking the discrete Fourier transform of a chord's chromagram and focusing on components F1 through F6. The magnitude and phase of each component describe the strength and orientation of a particular musical interval type, giving insight into the way that chord sounds. For our study, we used binary chromagrams.

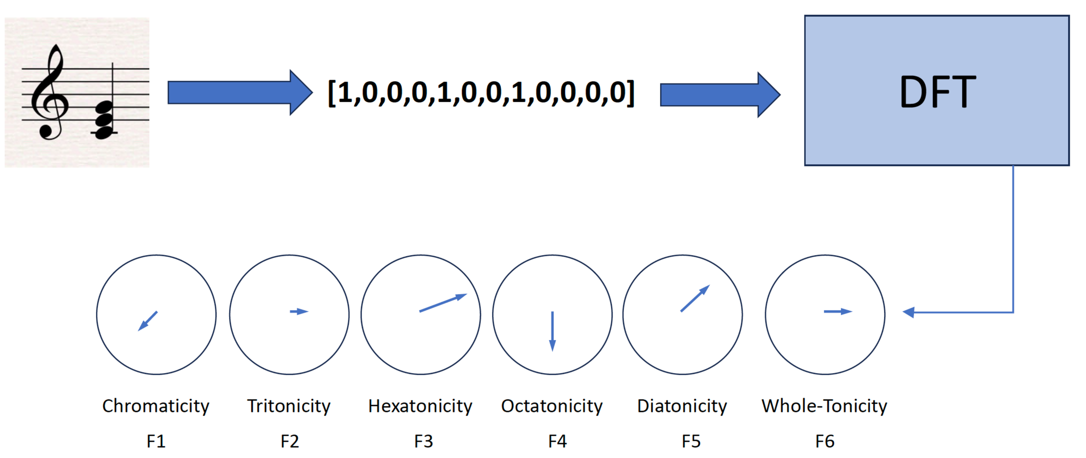

Fig. 1: Tonal Interval Vectors: quantifying intervallic properties of chords

PMD: Measuring the distance and direction between TIVs

We present Polar Manhattan Displacement, a multi-dimensional extension of the L1 Manhattan distance applied to pairs of magnitude and phase values. PMD measures the pairwise distances between the Fourier components of two TIVS, thus describing the differences between the chords in terms of their musical interval content. PMD is directional, so encodes the order of chord transitions, and is multi-dimensional, so it describes each intervallic difference separately. Significantly, the overall PMD between two chords remains consistent regardless of which chords (or how many other chords) have occurred in between them in a sequence. This allows us to numerically compare different versions of similar chord sequences.

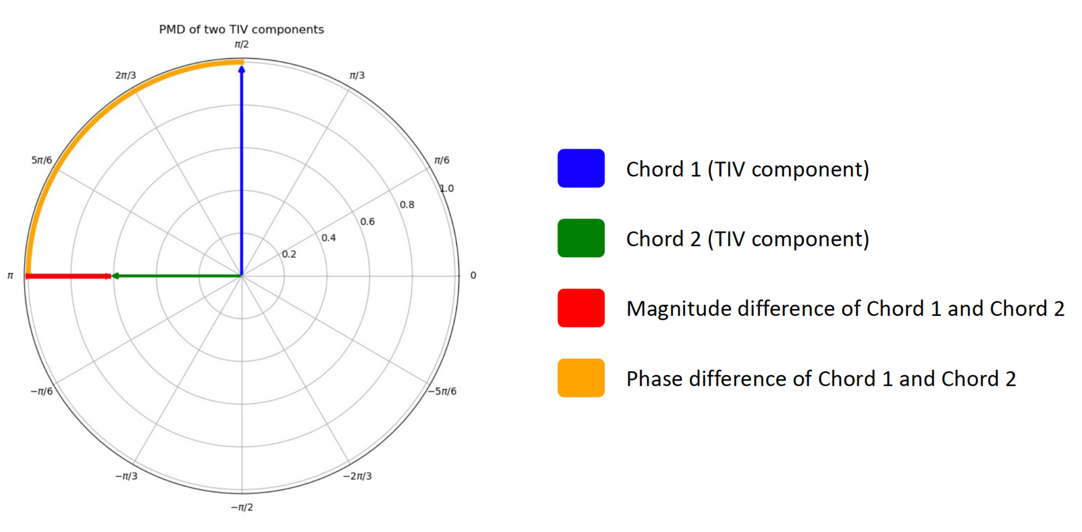

Fig. 2: Components of PMD: ∆ Magnitude and ∆ Phase

Additive Properties of PMD

The horizontal axis represents the movement from one chord type to another, and the vertical axis represents transpositional movement from one root note to another. Each darkened point represents a chord's TIV, and each arrow represents the displacement between two TIVs.

The fat arrow represents the PMD from a G dominant 7 chord directly to a C major chord. Notice that it is directional, like the chord transition itself. The smaller arrows represent two chord sequences that travel from G7 to C major. If we sum the PMDs of either sequence, they will equal the displacement directly between the two original chords. Thus, the overall displacement remains the same regardless of variations in the chord sequences.

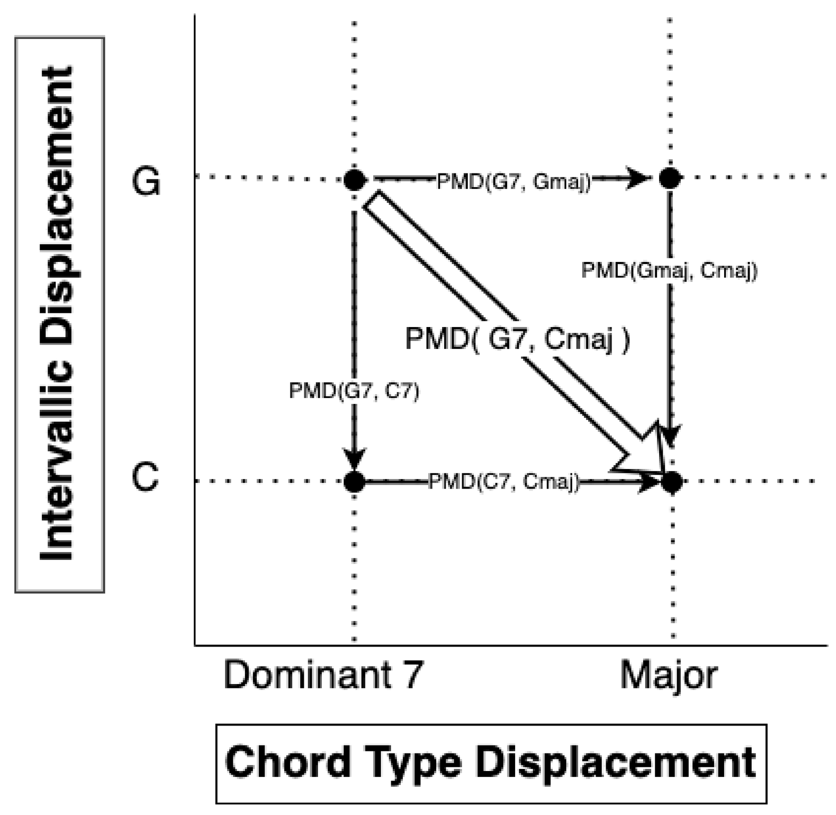

Fig. 3: PMD additive properties and type/transposition decoupling

Exploring Harmonic Relationships: Tritone Substitution

Musicians often substitute certain chords for one another to provide harmonic variety. For example, it is common practice to replace a dominant 7th chord with the dominant 7th whose root lies an augmented 4th away. This is known as a tritone substitution. PMD can provide an objective means of numerically describing and analysing the tonal properties of such relationships. Furthermore, all tritone substitution pairs have the same displacement value, allowing us to identify the relationship from its PMD alone.

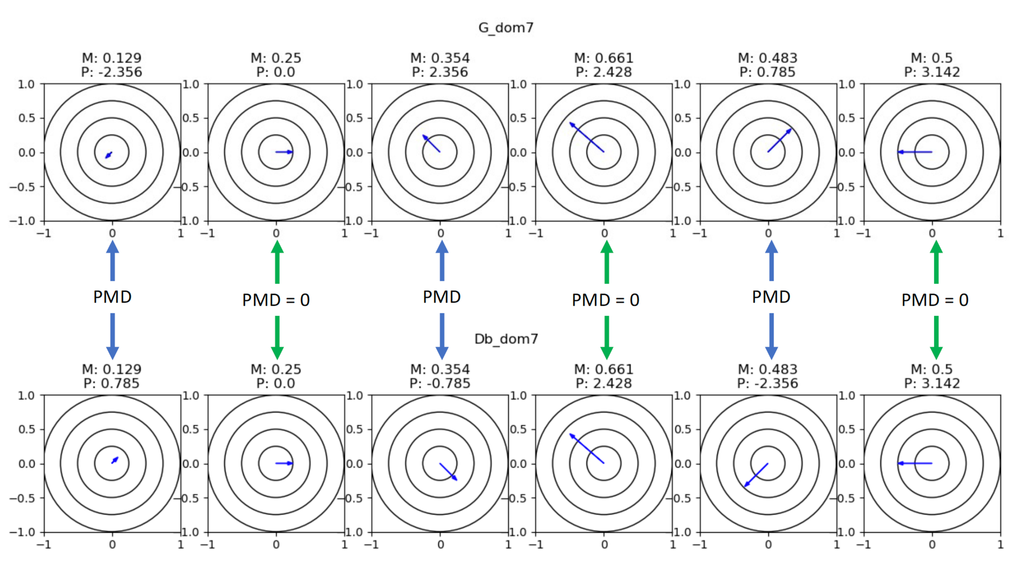

Fig. 4: PMD of a Tritone Substitution Pair

Identifying Chord Progressions with PMD: V7-I

Many MIR tasks could benefit from the ability to autonomously identify certain chord progressions, particularly those which are harmonically significant, such as the V7 – I progression. Traditionally, identification can be hampered by issues such as missing chord labels, inconsistent chord spelling, and differences of transposition. In most cases, a particular chord progression can be identified by its unique PMD value, regardless of transposition and without the need for any chord annotation. For example, any V7 – I progression will have the same PMD, regardless of the key involved.

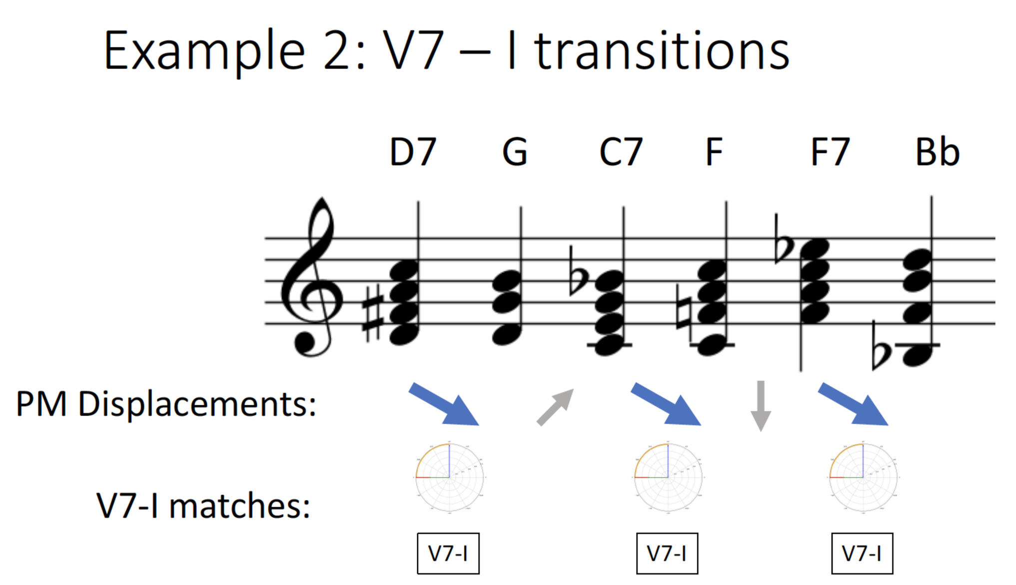

Fig. 5: Identifying V7-I type progressions with PMD Pair

PMD values can be chained together to represent and compare different chord sequences

PMD values can be added together to represent and measure the individual and overall tonal displacements of chord sequences. Here, we compare two alternative harmonisations of the first four bars of 'Georgia On My Mind'. Although the two versions have different chord content and harmonic rhythm, PMD measures the displacement of each individual transition, as well as the larger harmonic displacements of each sequences. The overall displacement of each example is the same.

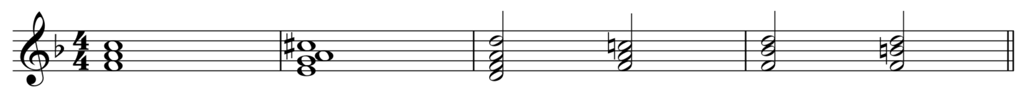

Fig. 6: GOMM version 1

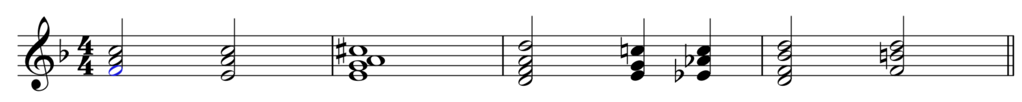

Fig. 7: GOMM version 2

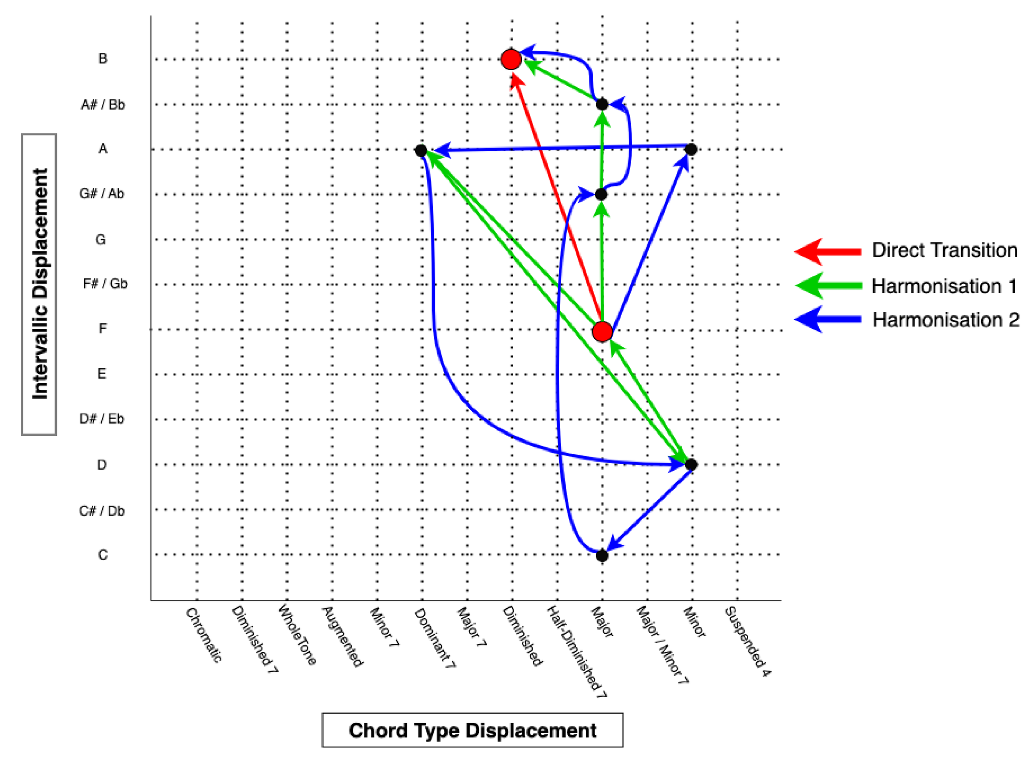

Fig. 8: PMD map of two harmonisations of Georgia On My Mind

Future Work

- Extension to accept audio domain input
- Incorporation into Chord Sequence Graph schema[5]
- Automated analysis of chord sequences and harmonic patterns in large musical corpora

Possible Applications

- Computational musicology
- Music Similarity rating
- Content-aware music recommendation
- Objective contributions to authorship / plagiarism investigations

References

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[5] Jeff Miller et al. "Discovering Common Practice: Using Graph Theory to Compare Harmonic Sequences in Musical Audio Collections". In: *ACM International Conference Proceeding Series.* New York, NY, USA: ACM, July 2021, pp. 93–97.

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